The notion of de-ritualization as a lens through which to view the teaching and learning of (secondary and university) mathematics

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Olov Viirman

Department of education

Uppsala University

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Mathematics as discourse

- Mathematics and mathematics teaching and learning can be viewed as discursive activities
- Doing mathematics is seen as engaging in mathematical discourse
- Mathematical objects are seen as discursively constituted, that is, they obtain meaning and existence through mathematical discourse

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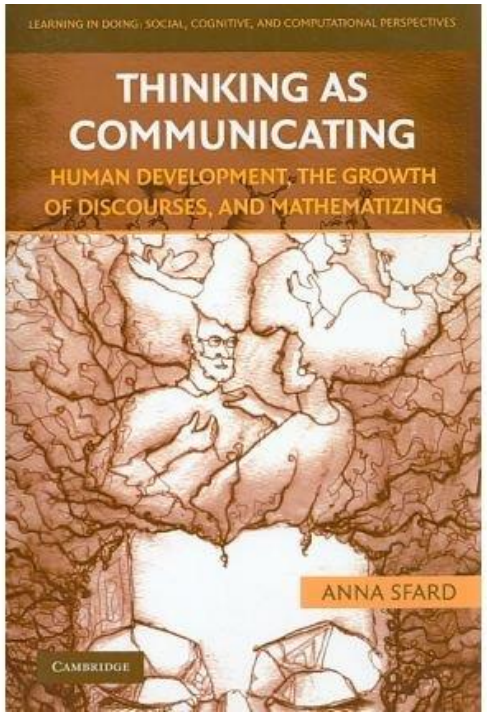
Commognition (Sfard, 2008)

- Basic tenet: “patterned, collective forms of distinctly human forms of doing are developmentally prior to the activities of the individual”
- Thinking: an individualized version of (interpersonal) communicating
- Types of communication, “set apart by their objects, the kinds of mediators used, and the rules followed by participants and thus defining different communities of communicating actors” are called *discourses.*
- Learning is development of discourse. Individual learning is individualization of discourse, becoming more capable of participating in the discourse, with others as well as with yourself



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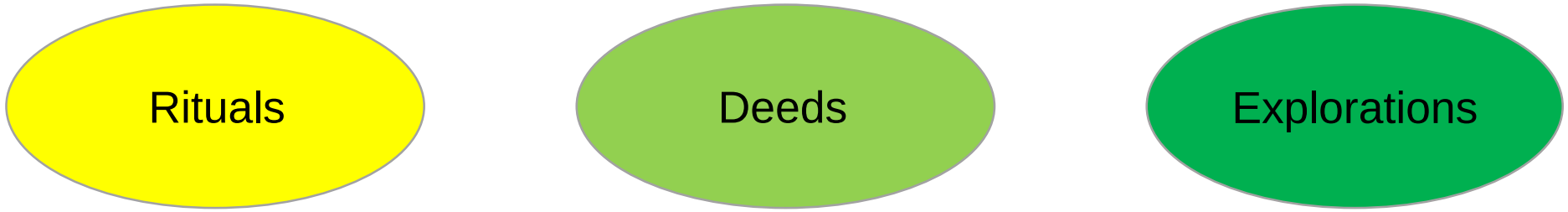
Characteristics of discourses

- Word use
  - Words specific to the discourse of everyday words used in discourse-specific ways, e.g., function, proof, Riemannian manifold
- Visual mediators
  - Visual objects operated on as part of the discursive activity, e.g., graphs, symbols
- Narratives
  - (Sequences of) utterances describing objects, their relations and/or processes upon them, subject to endorsement or rejection within the discourse, e.g., definitions, theorems, formulas
- Routines
  - Repetitive patterns characteristic of the discourse, e.g., algorithms, proof techniques

Routines

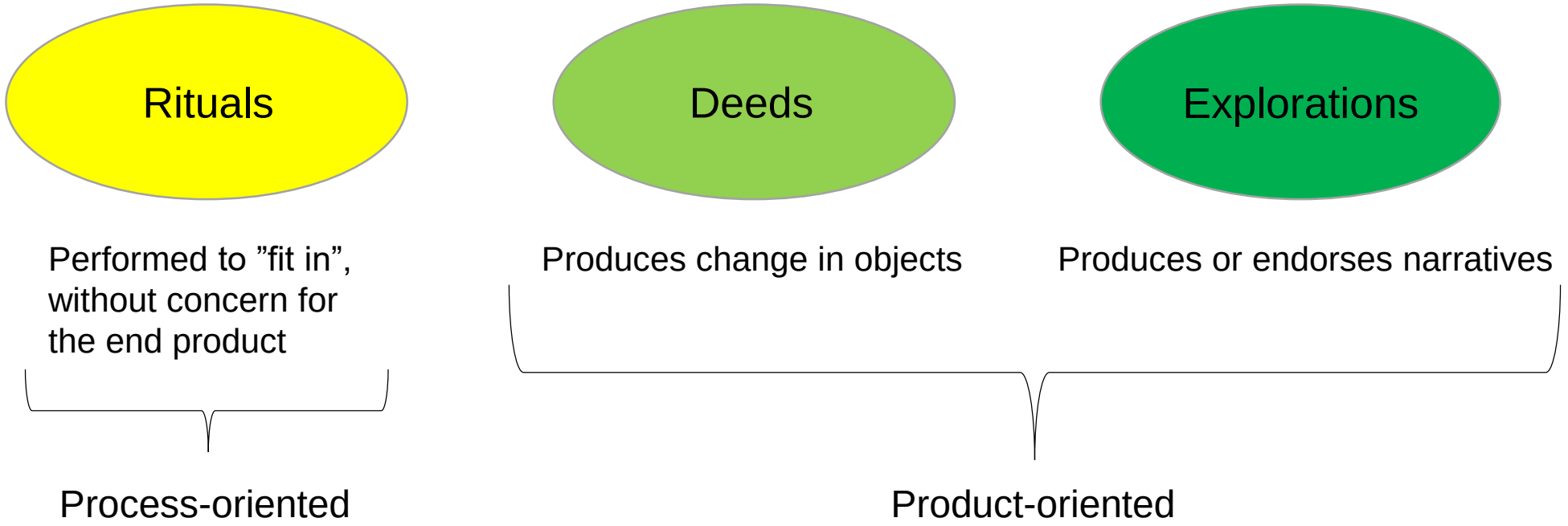


Performed to ”fit in”, without concern for the end product

Produces change in objects

Produces or endorses narratives

Routines


Rituals
Deeds
Explorations
Performed to ”fit in”, without concern for the end product
Produces change in objects
Produces or endorses narratives
Process-oriented
Product-oriented

Routines – further theoretical development

- Lavie, Steiner & Sfard (2019) attempt to give an operationalized definition of ’routine’, building on the notions of ’task situation’, ’task’ and ’procedure’
- In simple terms, a routine is conceptualized as a *task-procedure pair*
- One important consequence of this work is that the notion of routine becomes personalized and contextualized
- It also places the idea of ’routine’ at the very center of any discursive activity
- Indeed, Lavie et al claim that *learning is routinization*

- But, what then happens to rituals and explorations?

B1: Just multiply it with k?
R: k times what?
B2: Times R_n.
R: Exactly.
(…)
R: This is, you multiply the rate with the previous population. This is, you know, like, let's say that this is the amount of the increase, not in total. Can you see my point?
B4: So we have to move it from…, to the other side?

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Ritual vs exploration

- Explorations
  - Deal with mathematical objects and their realizations
  - Flexible routines, narratives built upon previously established discourse
  - Rely on internal authority and the rules of the discourse
  - Focus on the mathematical narratives produced or substantiated
- Rituals
  - Deal with symbols unrelated to the mathematical objects
  - Rigid routines weakly connected to previously established routines
  - Rely on external authority
  - Focus on the steps and procedures of the activity
- Rituals (process-oriented routines) and explorations (product-oriented routines) are thus no longer seen as opposites, but rather as endpoints on a continuum
- Ritual often a necessary part of learning

De-ritualization

- The process of transforming ritual to exploration is called de-ritualization

“an observed change in the performance will count as a step in the de-ritualization of the respective routine only if it indicates that the performer’s attention shifted from the performance as such to its outcome” (Lavie et al, 2019, p. 167)

- Lavie et al describe a number of characteristics of such changes

Characteristics of de-ritualization

- Flexibility
  - Increased ability to perform a task in different ways
  - Example: solving a quadratic equation algebraically and graphically
- Bondedness
  - reduced redundancy in procedures performed; all steps necessary to achieve the outcome
  - (x-2)(x-3)=0
- Applicability
  - Increased range of tasks for which the routine is recognized as useful
  - Using quadratic equations to solve problems about right triangles

Characteristics of de-ritualization

- Agentivity
  - Increased responsibility fo decisions taken in performing the routine
  - ”what is needed to solve this problem” rather than ”how did the teacher solve this problem in class yesterday”
- Objectification
  - Increased ability to describe the routine in terms of objects rather than processes
  - ”These numbers satisfy the quadratic equation” rather than ”We applied the quadratic formula and this is what we got”
- Substantiability
  - Increased ability to substantiate one’s actions (within the discourse)
  - ”Through completing the square one can derive the quadratic formula” rather than ”yesterday the teacher showed us this formula for solving quadratic equations”

Example – mathematical modelling in biology

- A cross-disciplinary project involving mathematics and biology
- 12 first-semester biology students, four three-hour meetings (concurrent with mandatory math course)
- Centred around groupwork on mathematical modelling problems with biological content
- Groups of 3-5 students

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The ”Terror bird” task

Estimate the weight of an extinct species of bird using data on the relationship between femur circumference and body weight among various bird species

Table 1
Terror Bird; femur circumference and body weight of birds (adapted from Giordano et al., 2013).

Femur circumference (cm)	Body weight (kg)
0.7943	0.0832
0.7079	0.0912
1.000	0.1413
1.1220	0.1479
1.6982	0.2455
1.2023	0.2818
1.9953	0.7943
2.2387	2.5199
2.5119	1.4125
2.5119	0.8913
3.1623	1.9953
3.5481	4.2658
4.4668	6.3096
5.8884	11.2202
6.7608	19.95
15.136	141.25
15.85	158.4893

Students were expected to draw on and adapt a previous solution to up a model and then use a graphical solution to come up with an estimate

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Traces of de-ritualization in student work

A1: If we look at this big bird here, then this [points at final entry in the table] is 15 cm, but the whole bird weighs 158 kg. And what weighs is this part here.
A2: Yes, the body.
A1: And it is like a ball.
A3: Yes, more or less.
A4: The head is pretty heavy too.
(…)
A1: Never mind the head. If we just try to do what he said. Circumference to the third times some constant or other.

- This group mimicked the previous solution (”If we just try to do what he said”) but adapted it to the current problem, showing signs of increased applicability
- However, they were not able to adapt the graphical solution model, because the data set was too confusing

Traces of de-ritualization in student work

C4: Look here, there are two individuals with exactly the same [femur circumference], and one weighs half of the other.
C3: Wow.
(
C4: This doesn't make sense. Look here, some of them have smaller thighbones than the others and still they weigh twice as much.
C3: This one has just as large a thighbone as the other, but there is a very large difference in weight.
C4: Look here. These two, the same number, but widely different weights.
C2: Yes, that's why we need to find a line.
C4: What? To find the most correct curve?
C1: The best possible.

- This group managed to adapt the graphing technique to the ”messy” dataset, thus displaying signs of increased applicability and agentivity
- However, they did not manage to adapt the model, instead choosing to work with a linear model

Tracing de-ritualization – further studies

- Lavie and Dvir (2023) and Steiner (2018) have studied de-ritualization in young children, for instance in terms of increased bondedness
- Nachlieli and Tabach (2022) studied classroom learning as a de-ritualization process in the context of elementary school teacher education
- They developed a tool for detecting signs of de-ritualization in student activity

Flexibility		Are the Gauss procedure steps modified to solve the given SoS question? That is, does this procedure contribute to the CPSS?
Bondedness		Do the students regard the output of a previous procedure step as input to the next?
Substantiability		Do students substantiate their performance and outcome - - On their own? - In response to others?
Students' Agentivity	**Presenting student**	Do presenters provide a complete solution on their own? Do presenters freely address other students' questions about the solution?
	Other students	Do other students react or ask questions about the presented solution? Do they address the instructor or the presenting student?

- This tool looks beyond the activity of the individual student
- Partly tailored to the specific learning situation, it could be modified for use in other settings

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De-ritualization in teaching

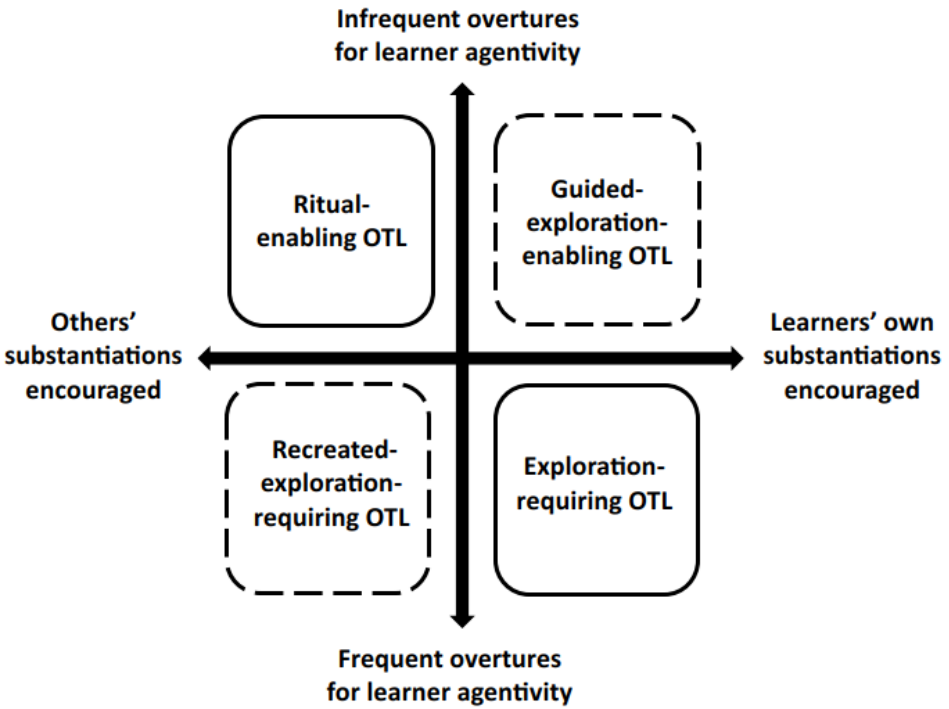
- Österling (2022) operationalized de-ritualization as an analytical tool for studying “teaching-as-usual”, drawing on data from the practicum of two prospective secondary mathematics teachers

- Presents a set of de-ritualizing moves connected to the characteristics of de-ritualization
- However, these are perhaps a bit too generic to be practically useful

De-ritualising moves	Descriptions
Bondedness	Encourage turning a sequence or previously disparate steps into a new compound procedure through focusing the connections between steps.
Flexibility	Encourage learners to find more than one way of performing a task.
Substantiability	Encourage substantiation of results beyond the steps performed as procedure.
Applicability	Encourage the extension of task situations for previously known procedures.
Learner agentivity	Encourage learner agentive participation, e. g. to decide for themselves what the task is, what to do, and if a procedure worked or not.
Objectification	Encourage discussions of what characterises objects (rather than how to use it), or legitimise objectification in explanations.

De-ritualization in teaching – cont'd

- Building on Nachlieli and Tabach's (2019) notions of ritual-enabling and exploration-requiring opportunities to learn (OTL's), Christensen et al.(2023) developed what they labelled *hybrid OTL's,* focusing on substantiability and agentivity
- While interesting, this approach can quickly become unwieldy
  - Six characteristics of de-ritualization means $2x2^6$ potential hybrid OTL's



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Task design for de-ritualization

- In Viirman and Jacobsson (2023) we present a set of strategies for modifying standard exercises to promote de-ritualization
- Flexibility
  - Solve the exercise in (at least two) different ways
  - Describe your solution using different representations
- Bondedness
  - Explain how the different steps in your solution contribute to it and how. Which steps are necessary? Which are specific to this particular exercise?
  - Compare a set of solutions to similar exercises and abstract a general procedure from them
- Applicability
  - Construct an exercise which can be solved using the same procedure you just used
  - How can you modify the given exercise and still be able to solve it using the same procedure?
  - Students can also be given seemingly unrelated follow-up tasks that can be solved using the same procedure

Task design for de-ritualization

- Agentivity
  - Gradually decreasing scaffolding
  - Solve the exercise using a solution strategy of your choice, and then justify your choice
- Objectification
  - Describe the solution you just obtained using geometric/algebraic terminology
  - Generally, asking students to present their solutions in terms of the mathematical objects rather than just the symbols
- Substantiability
  - Provide mathematical arguments for why the steps in your solution work
  - Prove that the claim you make is correct

Task design for de-ritualization - examples

Multivariable calculus

- Standard exercise:
  - Compute the directional derivative of the function $f(x, y) = e^{y+2x}$ at the point $a = (2,1)$ in the direction $u = (3,4)$.
- Modification:
  - Do this first by calculating the gradient $\nabla f(a)$, forming $\hat{u} = \frac{u}{|u|}$ and calculating $\nabla f(a) \cdot \hat{u}$, and second by setting $r(t) = a + t\hat{u}$ and $g(t) = f(r(t))$, and calculating $g'(0)$.
  - Now explain why these two procedures always give the same answer

- We argue that this modification supports flexibility and substantiability

Task design for de-ritualization - examples

Functions and equations

- Standard exercise:
  - Consider the functions $f(x) = x + 2$ and $g(x) = 6 - 2x$. If you multiply them you get a quadratic function $h(x)$. Determine where the graphs of all three functions cross the coordinate axes.
- Modification:
  - Now do the same for two other linear functions f and g.
  - Based on the results from these two exercises, formulate a conjecture about what relationship might hold between the crossing points with the coordinate axes of two general linear functions f and g and their product h.
  - Prove your conjecture.

- This modification supports bondedness and substantiability

Ongoing work

- Whether these strategies actually DO promote de-ritualization remains to be investigated
- Together with a mathematician colleague I teach an ”advanced course in mathematics and didactics of mathematics”, aimed at prospective upper secondary mathematics teachers
- Here, we try (among many other things) to design activities promoting de-ritualization in our students
- We hope to secure funding for researching the development of the students’ mathematical discourse during the course
- This would include signs of de-ritualization

Thank you for your attention!

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