## Developing and assessing mathematics teacher noticing with vignettes as representations of theory and practice

#### Marita Eva Friesen

Today there is a broad consensus that teacher noticing forms an essential part of teacher expertise and of what makes the teaching profession distinctive. Consequently, what mathematics teachers attend to in classroom situations, how they interpret it based on their professional knowledge and how noticing can be learned are important topics in teacher education, professional development and related research. Vignettes as representations of practice (e.g., in the form of video clips or cartoons) are widely used for developing and assessing teacher noticing in a systematic way close to classroom practice. However, vignettes used in noticing research do not only represent teaching practice: The classroom episodes are often purposefully constructed to represent selected aspects of mathematics education theory that can be related to particular norms or quality aspects of mathematics instruction. The talk will present examples from vignette-based studies on teacher noticing along with key considerations for the design of different vignette types depending on underlying research goals and/or learning objectives. In particular, the potential of vignettes showing high-quality teaching vs. vignettes showing breaches of the instructional norm will be highlighted and discussed for developing and assessing noticing in mathematics teacher education and professional development.

# Young children's approaches to mathematics: Inquiring into mathematics through bodily, multimodal processes.

#### Martin Carlsen

Mathematics education research has since the beginning of the 2000s shifted from a more cognitive view of learning to a more sociocultural view of learning. The so-called inquiry-based approach to mathematics teaching and learning has thus been characterized by dialogue, heavily emphasizing verbalizations and questions. In this talk I will argue that the emphasis on verbal communication has been too strong, at least when researching young children and their mathematical inquiries. My aim is to broaden the perspective of how mathematical inquiries manifest themselves when children engage in and participate in mathematical activities. This will be done through a re-conceptualization of mathematical inquiry as to also encompass bodily and multimodal cognition.

## The notion of de-ritualization as a lens through which to view the teaching and learning of secondary and university mathematics

#### Olov Viirman

Over the last decade, the theory of commognition (Sfard, 2008), with its view of mathematics as a discursive practice, has seen increased use in research on the teaching and learning of mathematics at the secondary and tertiary level. In this talk, I will take as a starting point the commognitive idea of mathematics as a routine-driven activity. Sfard distinguishes between process-driven routines, rituals, and product-driven routines, explorations, which are viewed not as opposites but as

endpoints on a continuum. I will pay particular attention to the notion of de-ritualization, that is, discursive activity aimed at shifting routine use away from the ritualised and towards the exploratory end of the spectrum. Examples will be drawn from my own research on, for instance, students' group work on mathematical modelling tasks; mathematicians and mathematics educators team-teaching mathematics and the didactics of mathematics to prospective upper secondary teachers; and task design promoting de-ritualization; but also on the work of other commognitive researchers.

## Decision making and research project design: an example from the work with Interactive Theorem Provers

#### Paola Iannone

All those who have conducted research projects in mathematics education know that research does not proceed in the linear fashion in which it is presented in research papers and talks - especially qualitative research. In this talk I will focus on the decision-making process that happened during the design, data collection and analysis of one of my research projects by annotating one of the research talks I have given about the project.

The original abstract of the talk I will be discussing was as follows:

In this talk, I present results from a study investigating the potential impact of undergraduate mathematics students' engagement with an interactive theorem prover (LEAN) on their proof production. The students were offered the opportunity to interact with LEAN in optional evening sessions throughout the first semester of the first year and were also shown the potential of this software in some of the lectures. The participants, whose experiences with LEAN varied, were interviewed at the end of the teaching period and were asked to prove seen and unseen mathematical tasks using a think-aloud protocol.

Here we focus on the proofs produced by thirty-six students on an unseen task. We first scored the proofs of both experienced and non-experienced LEAN. We then analysed qualitatively the proof writing alongside the interview transcripts following the Fukawa-Connelly (2012) framework. Results suggest that engagement with LEAN assists in making the proof-writing more rigorous; helps with the deployment and use of definitions in proofs; and with the syntax of proof writing. Finally, we will discuss challenges that prevented students from engaging with LEAN.

## Supporting collective looking-back practices: Who-Is-Right tasks

### Michal Tabach

Engaging with problem solving in mathematics, one goes along the following four stages: understanding the problem; devising a plan; enacting the plan; and looking back (Pólya, 1945/1973). The looking-back stage is rarely observed in students' problem solving in spite of its recognized importance. The importance of this stage is attributed to practices of engagement with queries on verification of the obtained solution(s), comparative consideration of alternative solutions, and formulation of implications for future problem solving. I refer to such practices as looking-back practices. In the talk I will demonstrate how the looking-back practices can be

evoked in small-group classroom discussions of controversial worked-out solutions to word problems. Such tasks are known as who-is-right tasks. The data consisted of audio- and videotapes of six small groups of high-school students working on a who-is-right task in the context of percentage. The data analysis, informed by a discursively-oriented perspective on problem solving, attended to strategies, dialogical moves and mathematical resources enacted by the students towards attempted agreement as to which of the solutions should be endorsed and why. The findings imply that who-is-right tasks have potential for supporting collective looking-back practices. The study contributes to our knowledge on enactment of mathematical resources in problem-solving discourse and on patterns of students' dialogic participation in small-group problem solving.

# The visible and the invisible in mathematics education research: tales of arguments, signs, and disciplinarities

#### Andreas Moutsios-Rentzos

Mathematics education research concerns a series of decisions within the complexity of the educational phenomena. The theoretical frameworks and the scientific methodologies intertwined with the technological means are the lenses that constitute the phenomenon we research. Depending on the theoretical, methodological and technological tools we choose, we choose which phenomenon we investigate, which aspects of the educational complexity are (directly or indirectly) visible for our research, our scientifically acceptable investigations and inferencing. In this talk, I shall consider argumentation, mathematical notation and digital storytelling as means for highlighting aspects of the role of the theoretical-methodological-technological lenses in mathematics education research. Both intradisciplinary and interdisciplinary aspects of the lived complexity will be considered.

### Introducing the role of being an advocate in mathematics teacher education

#### Tamsin Meaney

Mathematics teacher education is often described in terms of the mathematical content and pedagogy that teachers need. However, recent calls for equity in mathematics education demand a broadening of this view. In this presentation, we articulate a theoretical description of what the role of being an advocate in language-diverse classrooms could involve and some of the practical challenges that mathematics teacher educators may have when introducing it, using empirical examples from our teacher education courses for teachers of Grades 1-7 in Norway. In the theoretical description of the role of being an advocate into mathematics teacher education, we distinguish between in-class advocacy and advocacy beyond the classroom and what these different kinds of advocacy might entail in language-diverse classrooms. The practical issues that we identified in raising different aspects of the role of being an advocate, in and outside of the classroom, in different cultural settings and how this knowledge could be used in teacher education to challenge preservice teachers' language ideologies and raise non-trivial issues.